

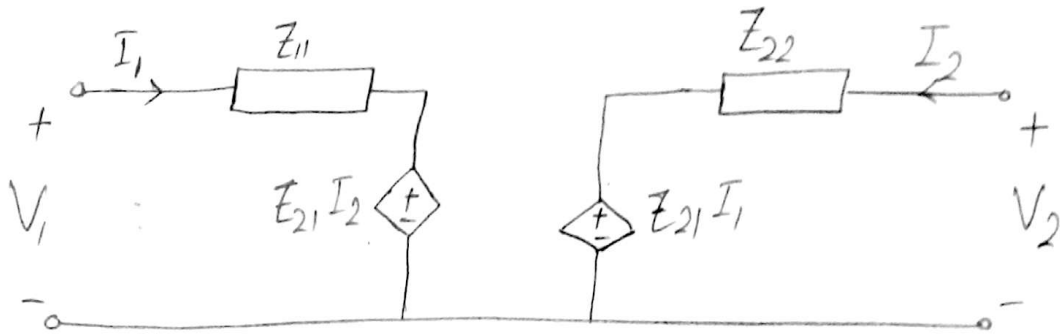
*Equivalent Circuit for different Parameters:-

1

1 Z-Parameters:

$$V_1 = Z_{11} I_1 + Z_{12} I_2$$

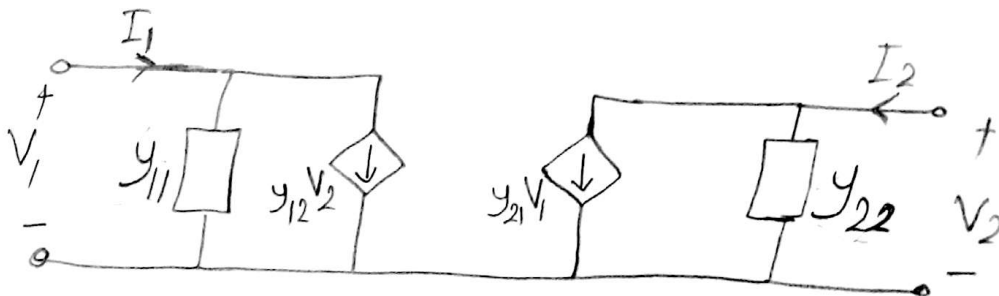
$$V_2 = Z_{21} I_1 + Z_{22} I_2$$



2 Y-Parameters:

$$I_1 = y_{11} V_1 + y_{12} V_2$$

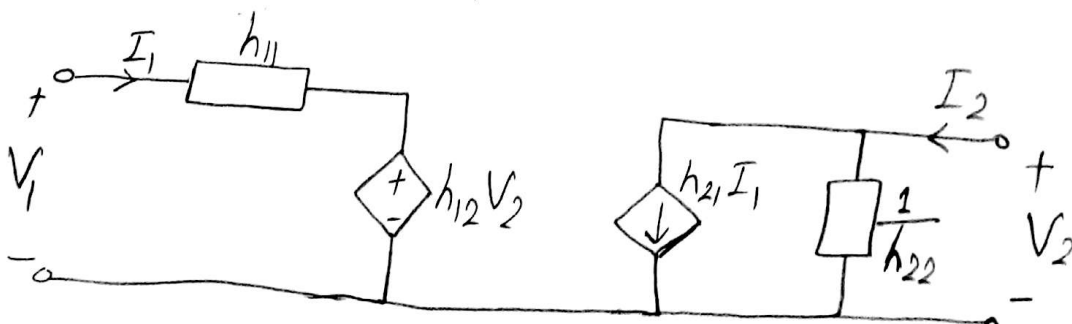
$$I_2 = y_{21} V_1 + y_{22} V_2$$



3 h-Parameters:

$$V_1 = h_{11} I_1 + h_{12} V_2$$

$$I_2 = h_{21} I_1 + h_{22} V_2$$



5

slants :-

* Relationship between models:-

[2]

[1] Relationship between Z-Param. & Y-Param.

$$\Rightarrow Z_{11} = \frac{y_{22}}{\Delta y}, \quad Z_{22} = \frac{y_{11}}{\Delta y}$$

$$Z_{12} = \frac{-y_{12}}{\Delta y}, \quad Z_{21} = \frac{-y_{21}}{\Delta y}$$

$$\Rightarrow y_{11} = \frac{Z_{22}}{\Delta Z}, \quad y_{22} = \frac{Z_{11}}{\Delta Z}$$

$$y_{12} = \frac{-Z_{12}}{\Delta Z}, \quad y_{21} = \frac{-Z_{21}}{\Delta Z}$$

$$-\Delta y = \begin{vmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{vmatrix}$$

$$-\Delta Z = \begin{vmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{vmatrix}$$

[2] Relationship between h-Param. & Z-Param. & y-Param. :

$$\Rightarrow h_{11} = \frac{1}{y_{11}} = \frac{\Delta Z}{Z_{22}}$$

$$\Rightarrow Z_{11} = \frac{\Delta h}{h_{22}}$$

$$\Rightarrow h_{12} = \frac{Z_{12}}{Z_{22}} = \frac{-y_{12}}{y_{11}}$$

$$\Rightarrow Z_{12} = \frac{h_{12}}{h_{22}}$$

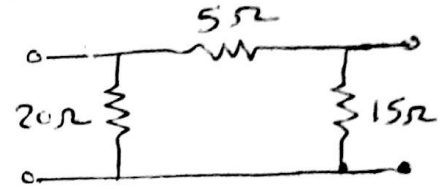
$$\Rightarrow h_{21} = \frac{y_{21}}{y_{11}} = \frac{-Z_{21}}{Z_{22}}$$

$$\Rightarrow Z_{21} = \frac{-h_{21}}{h_{22}}$$

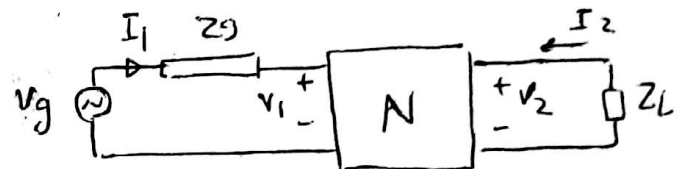
$$\Rightarrow h_{22} = \frac{1}{Z_{22}} = \frac{\Delta y}{y_{11}}$$

$$\Rightarrow Z_{22} = \frac{1}{h_{22}}$$

Q1: Find Z-Parameters for the circuit shown.



Q2: Find Z_{in} , current gain $\frac{I_2}{I_1}$, $\frac{I_2}{I_g}$, voltage gain $\frac{V_2}{V_1}$, $\frac{V_2}{V_g}$



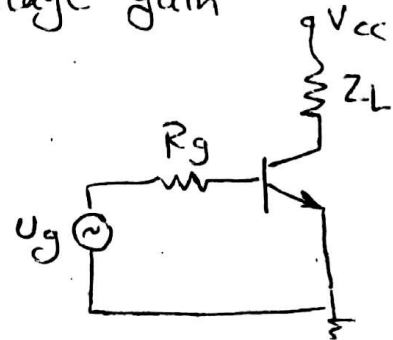
Q3: Compare between the equivalent circuit of the circuit given in Prob1 & using Z & h-Parameters.

Q4: Find Z_{in} , current gain and voltage gain (using h-model)

$$h_{fe} = 50 \quad h_{oe} = 25 \times 10^{-6} \Omega$$

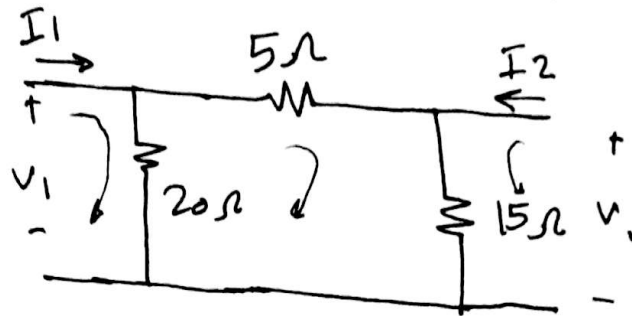
$$Z_L = 1000 \Omega \quad R_g = 1000 \Omega$$

$$h_{ie} = 1100 \Omega$$



Sheet * 2

Q1: Find Z Parameter for the circuit shown..



Solution

$$Z_{11} = \frac{V_1}{I_1} \Big|_{I_2=0} = (5+15) \parallel 20$$

$$\therefore \boxed{Z_{11} = 10 \Omega}$$

$$* Z_{12} = \frac{V_1}{I_2} \Big|_{I_1=0}$$

$$I'_2 = I_2 \times \frac{15}{25+15} = \frac{3}{8} I_2$$

$$\therefore V_1 = I'_2 \times 20$$

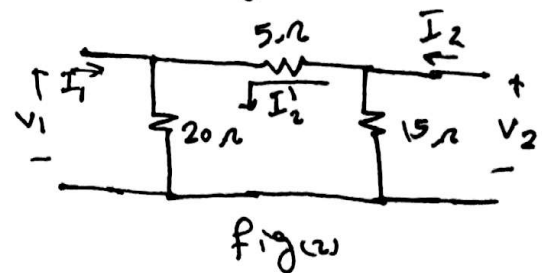
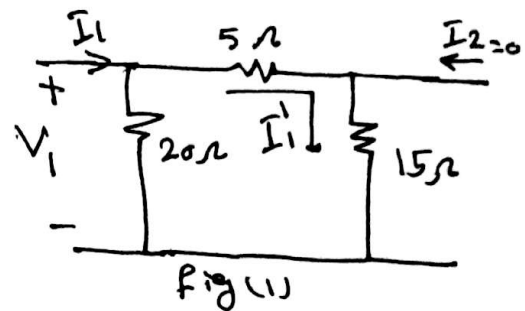
$$\therefore V_1 = \frac{3}{8} \times I_2 \times 20$$

$$\therefore Z_{12} = \frac{V_1}{I_2} = \frac{60}{8} = \boxed{7.5 \Omega}$$

$$* Z_{21} = \frac{V_2}{I_1} \Big|_{I_2=0}$$

From fig (1)

$$I'_1 = I_1 \times \frac{20}{20+20} = \frac{1}{2} I_1$$



$$\therefore V_2 = I_1' * 15$$

$$V_2 = \frac{1}{2} * I_1 * 15$$

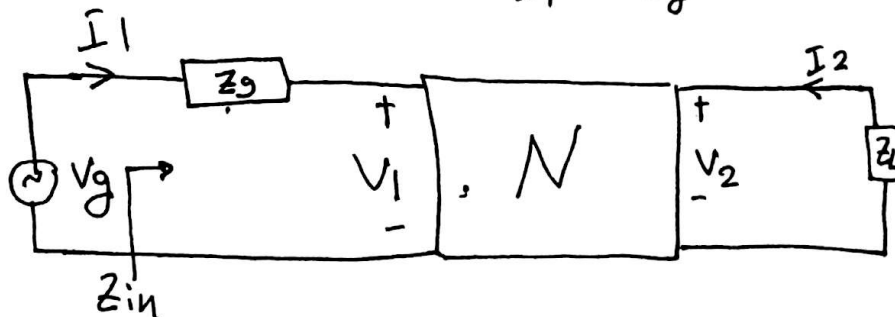
$$\therefore \frac{V_2}{I_1} = 7.5 \Omega$$

$$\therefore \boxed{Z_{21} = 7.5 \Omega}$$

$$\rightarrow Z_{22} = \frac{V_2}{I_2} \Big|_{I_1=0} \quad \text{from fig(2)}$$

$$\therefore Z_{22} = (5 + 20) // 15 = \boxed{9.37 \Omega}$$

2) Find Z_{in} , Current gain, $\frac{I_2}{I_1}$, $\frac{I_2}{I_g}$, voltage gain $\frac{V_2}{V_1}$, $\frac{V_2}{V_g}$



Solution

- $Z_{in} = Z_g + Z_{11}$, where Z_{11} = input impedance
- $\frac{I_2}{I_1} = h_{21}$
- $\frac{I_2}{I_g} = \frac{I_2}{I_1} = h_{21}$
- $\frac{V_2}{V_1} = \frac{1}{h_{21}} = g_{21}$

$$-\frac{V_2}{V_g} = \frac{V_2}{V_1} * \frac{V_1}{V_g} = \boxed{\frac{1}{h_{12}} * \frac{h_{11}}{h_{11} + Z_g}}$$

3) compare between the equivalent circuit of the circuit given in Prob using Z and h-Parameters.

Solution

H-parameter for the circuit

$$* h_{11} = \frac{V_1}{I_1} \Big|_{V_2=0} = 5 // 20 = \boxed{4 \Omega}$$

$$* h_{12} = \frac{V_1}{V_2} \Big|_{I_2=0}$$

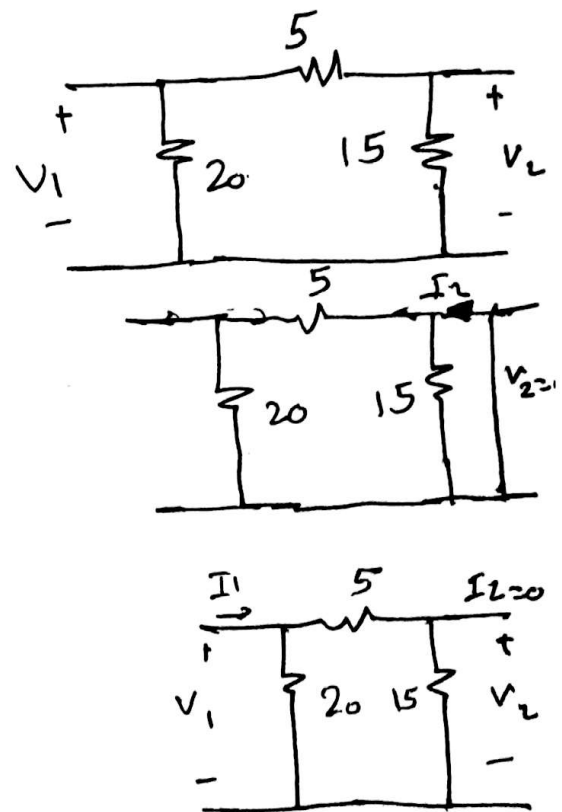
$$V_2 = V_1 * \frac{15}{15+5} = \frac{15}{20} V_1$$

$$\therefore \boxed{\frac{V_1}{V_2} = h_{12} = \frac{20}{15}}$$

$$\text{for } h_{21} = \frac{I_2}{I_1} \Big|_{V_2=0}$$

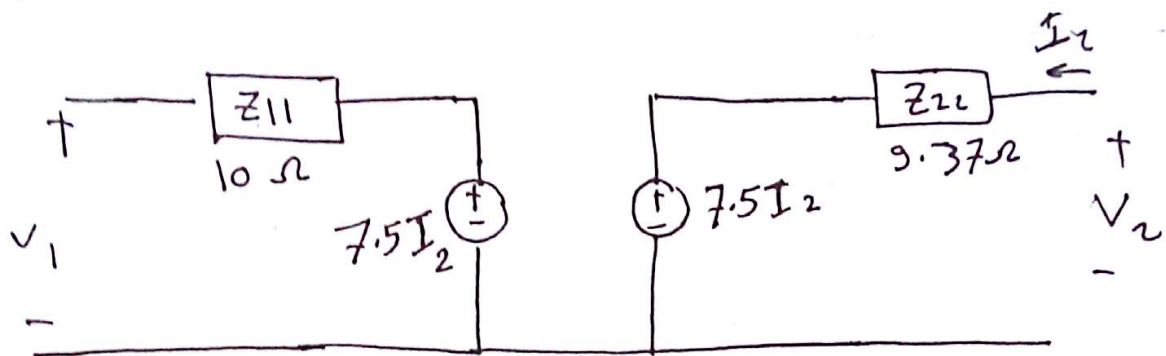
$$I_2 = -I_1 * \frac{20}{20+5} = -\frac{20}{25} I_1$$

$$\therefore \frac{I_2}{I_1} = -\frac{20}{25} = -1.25$$

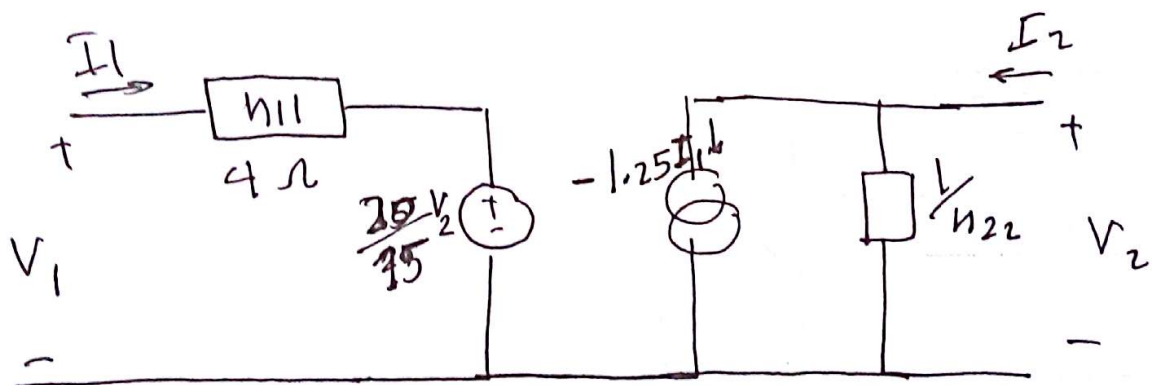


$$* h_{22} = \frac{I_2}{V_2} \bigg|_{I_1=0} = 25 // 15 = 0.106 \Omega$$

the equivalent circuit for Z-Parameter



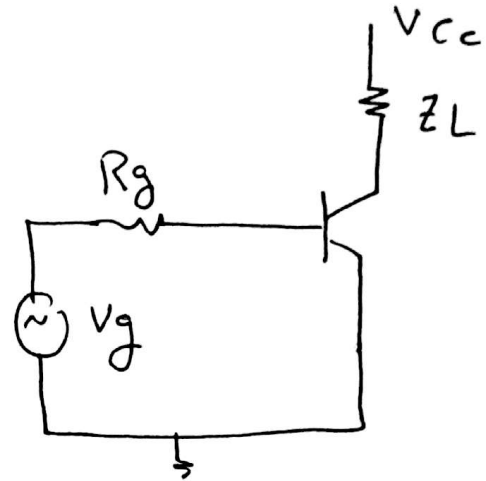
- The equivalent circuit for h-Parameter:-



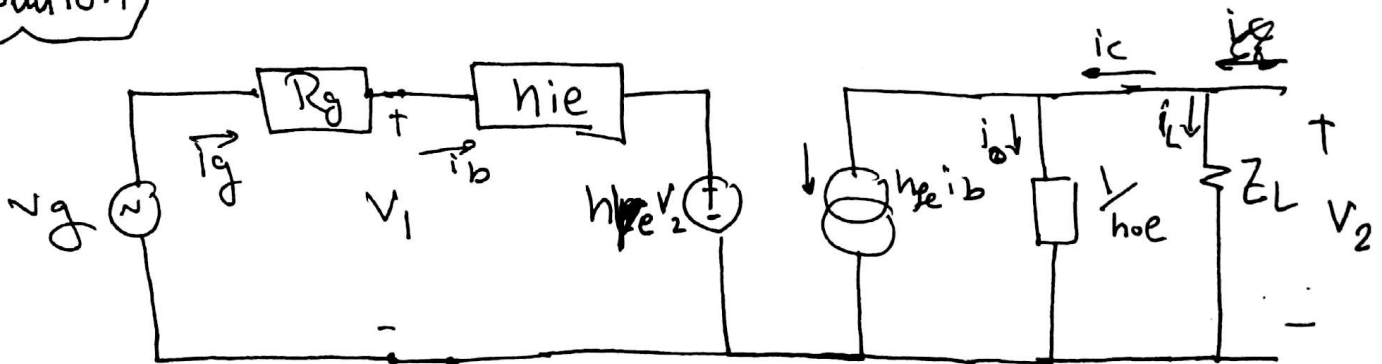
4) Find Z_{in} , Current gain and voltage gain using (h-Parameter)

$$h_{fe} = 50, h_{oe} = 25 \times 10^{-6} \Omega^{-1}, Z_L = 1000 \Omega, R_g = 1000 \Omega$$

$$h_{ie} = 1100 \Omega$$



Solution



$$* Z_{in} = \frac{V_g}{i_b}$$

$$i_b = i_g$$

$$V_g = i_g R_g + i_b h_{ie} + h_{re} V_2$$

$$= i_b R_g + i_b h_{ie} + h_{re} V_2$$

$$V_g = (R_g + h_{ie}) i_b + h_{re} V_2$$

$$V_g = (R_g + h_{ie}) i_b + h_{re} i_L R_L$$

$$= (R_g + h_{ie}) i_b + h_{re} i_L R_L$$

$$V_2 = + i_L R_L$$

$$i_L = -i_c$$

$$\therefore (V_g + R_g + h_{ie}) i_b - h_{re} i_c R_L \rightarrow \textcircled{1}$$

Find $A_i = \frac{i_L}{i_b}$

$$i_L = -i_c \quad \text{ve}$$

where $i_c = h_{fe} i_b + i_o$

$$i_c = h_{fe} i_b + V_2 h_{oe}$$

$$i_c = h_{fe} i_b + i_L R_L h_{oe}$$

$$i_L = -i_c$$

$$i_c = h_{fe} i_b - i_c R_L h_{oe}$$

$$\therefore i_c (R_L h_{oe} + 1) = h_{fe} i_b$$

$$\therefore A_i = \frac{-i_c}{i_b} = \frac{-h_{fe}}{1 + h_{oe} R_L} = \frac{-50}{1 + 25 \times 10^{-6} \times 1 \text{ K}} = \boxed{-48.78}$$

$$\therefore i_c = \boxed{-i_b A_i} \rightarrow \textcircled{1} \text{ معوضه}$$

$$\therefore V_g = (R_g + h_{ie}) i_b + (h_{re} A_i R_L) i_b$$

$$V_g / i_b = R_g + h_{ie} + h_{re} A_i R_L = \boxed{2098.8 \Omega} \#$$

$$A_v = \frac{V_2}{V_1}$$

$$V_2 = -i_c R_L = A_i i_b R_L$$

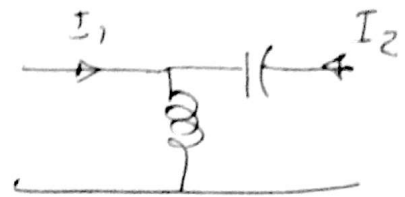
$$V_g = Z_{in} i_b$$

$$V_g = V_1$$

$$A_v = \frac{V_2}{V_g} = \frac{A_i R_L}{Z_{in}} = \boxed{-23.24}$$

Sheet #3

① Find A-Parameters



②

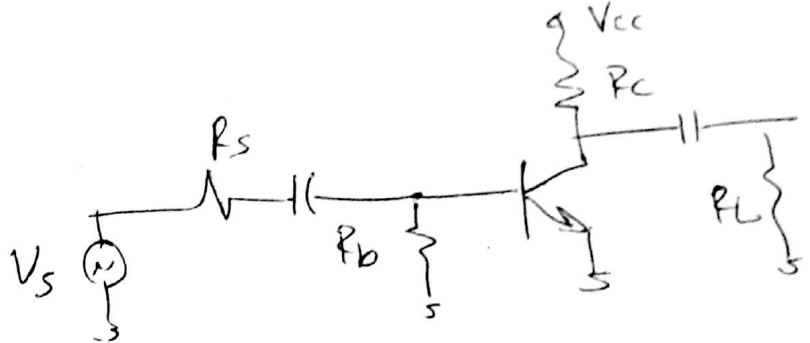
$$\text{hve } V_{ce} = 0$$

$$R_b = 1 \text{ K}\Omega$$

$$h_{ie} = 500 \Omega$$

$$h_{oe} = 50 \times 10^{-6} \text{ S}$$

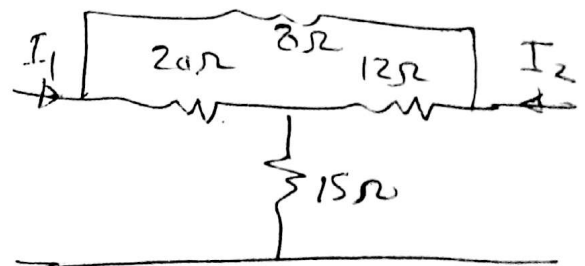
$$R_c = 15 \text{ K}\Omega \text{ } R_b = 1 \text{ K}\Omega$$



For the circuit shown; using h-model, Find Z_{in} , voltage gain, source voltage gain, current gain & Z_{out}

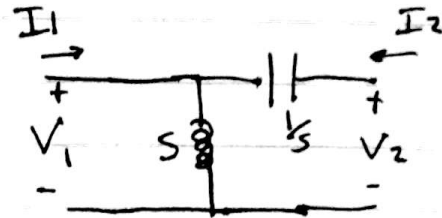
③ Find Short circuit impedance using Z-model

④ For the circuit shown, Find Z_{21} , Y_{11} , h_{22} .



sheet * 3

1) Find A - Parameters



Solution

$$\begin{aligned} V_1 &= A V_2 - B I_2 \quad \text{or} \quad V_1 = a_{11} V_2 - a_{12} I_2 \\ I_1 &= C V_2 - D I_2 \quad \text{or} \quad I_1 = a_{21} V_2 - a_{22} I_2 \end{aligned}$$

$$\therefore A = a_{11} = \left. \frac{V_1}{V_2} \right|_{I_2=0}$$

$$\therefore V_1 = V_2$$

$$\therefore \frac{V_1}{V_2} = 1$$

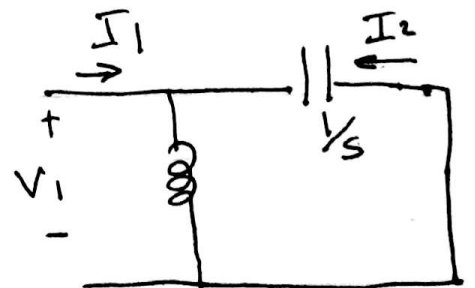
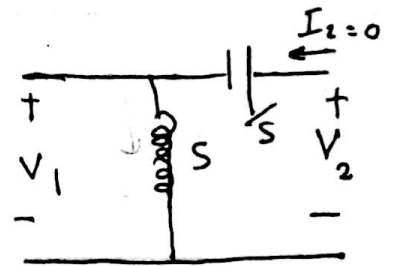
$$\therefore \boxed{a_{11} = A = 1}$$

$$\Rightarrow a_{12} = B = - \left. \frac{V_1}{I_2} \right|_{V_2=0}$$

$$V_1 = - I_2 \times \frac{1}{s}$$

$$\frac{V_1}{I_2} = - \frac{1}{s}$$

$$\therefore \boxed{a_{12} = \frac{1}{s}}$$



$$a_{21} = \frac{I_1}{V_2} \Big|_{I_2=0}$$

$$\therefore V_2 = I_1 * S$$

$$\therefore \frac{I_1}{V_1} = \frac{1}{S}$$

$$\therefore \boxed{a_{21} = \frac{1}{S}}$$

$$* a_{22} = -\frac{I_1}{I_2} \Big|_{V_2=0}$$

$$I_2 = -I_1 * \frac{S}{S+1}$$

$$\therefore \frac{I_2}{I_1} = -\frac{S}{\frac{S^2+1}{S}} = -\frac{S^2}{S^2+1}$$

$$\therefore \boxed{a_{22} = \frac{S^2+1}{S^2}}$$

2) For the circuit shown, using h-model, Find, Z_{in} , voltage gain, source voltage gain, current gain, Z_{out}

$$h_{re} V_{ce} = 0$$

$$R_b = 1 \text{ k}\Omega$$

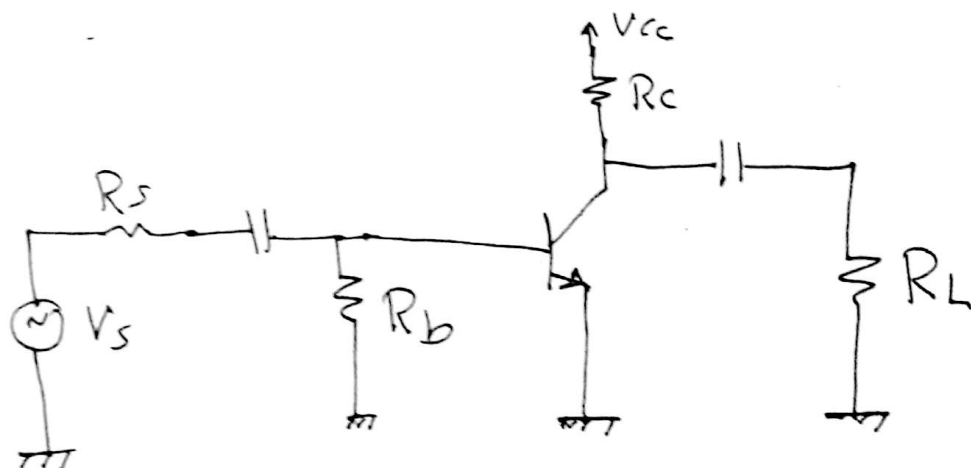
$$h_{ie} = 500 \Omega$$

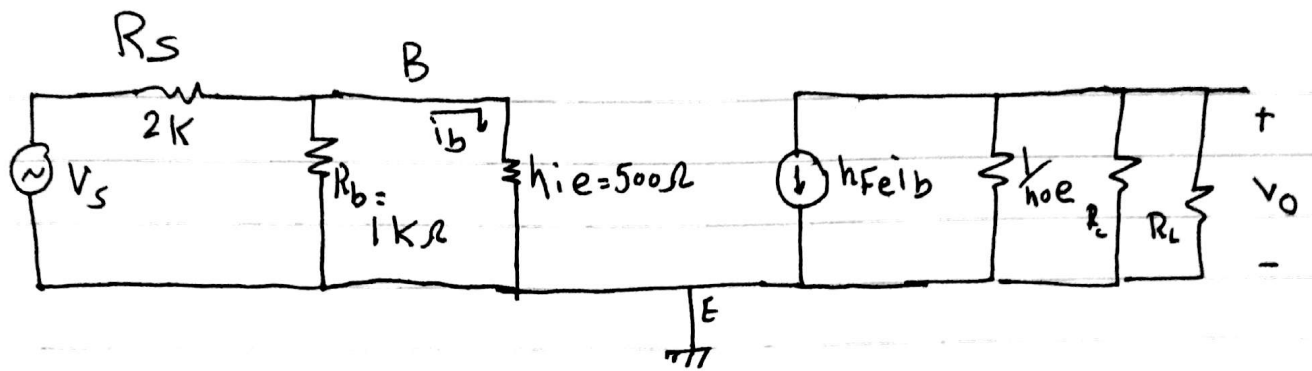
$$h_{oe} = 50 \times 10^{-6} \text{ S}$$

$$R_c = 15 \text{ k}\Omega$$

$$R_b = 1 \text{ k}\Omega$$

$$R_s = 2 \text{ k}\Omega$$





$$\textcircled{1} \quad Z_i = R_s + R_b \parallel h_{ie}$$

$$Z_i = 2k + 1k \parallel 500\Omega = 2k + 333\Omega = 2.3k\Omega$$

$$2- \quad A_v = \frac{V_o}{V_i}$$

$$\therefore V_o = -h_{fe} i_b \times Z_o \quad Z_o = R_L \parallel R_c \parallel \frac{1}{h_{oe}}$$

$$V_i = i_b h_{ie}$$

$$\therefore A_v = -h_{fe} \frac{Z_o}{h_{ie}}$$

$$3- \quad A_{v_s} = \frac{V_o}{V_s} = \frac{V_o}{V_i} \times \frac{V_i}{V_s} = A_v \times \frac{V_i}{V_s}$$

$$V_i = V_s \times \frac{h_{ie} \parallel R_b}{h_{ie} \parallel R_b + h_{ie}}$$

$$\frac{V_i}{V_s} = \frac{h_{ie} \parallel R_b}{(h_{ie} \parallel R_b) + h_{ie}}$$

$$\therefore A_{v_s} = -\frac{h_{fe} Z_o}{h_{ie}} \times \frac{h_{ie} \parallel R_b}{(h_{ie} \parallel R_b) + h_{ie}}$$

Current gain

$$A_i = \frac{i_o}{i_i}$$

$$i_o = \frac{V_o}{R_L}, \quad i_i = \frac{V_i}{h_{ie}}$$

$$\therefore \frac{i_o}{i_i} = \frac{V_o}{V_i} \times \frac{h_{ie}}{R_L} = A_v \frac{h_{ie}}{R_L}$$

$$\therefore A_i = A_v \times \frac{h_{ie}}{R_L}$$

$$5) Z_{out} = \frac{1}{h_{oe}} \parallel R_L \parallel R_c$$

3) Find short circuit impedance using Z-model:-

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4) for the circuit shown Find Z_{21} , y_{11} , h_{22}

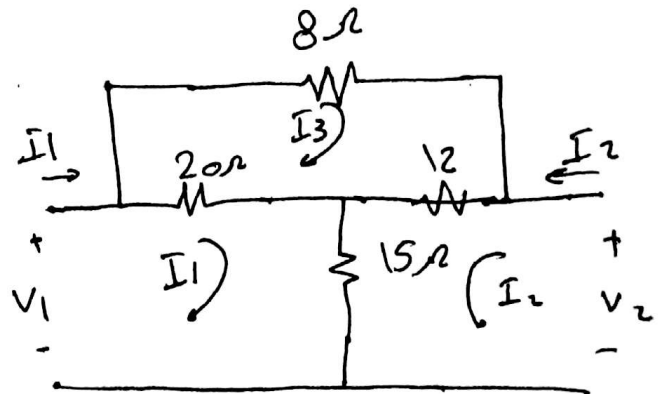
Solution

① Find Z-Parameter

Loop 1

$$V_1 = (20 + 15) I_1 + 15 I_2 - 20 I_3$$

$$V_1 = 35 I_1 + 15 I_2 - 20 I_3 \rightarrow \textcircled{1}$$



at Loop 2

$$V_2 = (12 + 15)I_2 + 15I_1 + 12I_3$$

$$V_2 = 15I_1 + 27I_2 + 12I_3 \rightarrow (2)$$

at Loop 3

$$0 = (12 + 8 + 20)I_3 - 20I_1 + 12I_2$$

$$0 = 40I_3 - 20I_1 + 12I_2$$

$$40I_3 = 20I_1 - 12I_2$$

$$\therefore I_3 = \frac{I_1}{2} - \frac{12}{40}I_2 \rightarrow (3)$$

from (3) into (1)

$$\therefore V_1 = 35I_1 + 15I_2 - 20 \left[\frac{I_1}{2} - \frac{12}{40}I_2 \right]$$

$$V_1 = 35I_1 + 15I_2 - 10I_1 + 6I_2$$

$$V_1 = 25I_1 + 21I_2$$

$$\begin{aligned} V_2 &= 15I_1 + 27I_2 + 12 \left[\frac{I_1}{2} - \frac{12}{40}I_2 \right] \\ &= 15I_1 + 27I_2 + 6I_1 - 3.6I_2 \end{aligned}$$

$$V_2 = 21 I_1 + 23.4 I_2$$

$$\therefore Z_{11} = 25 \quad Z_{12} = 21 \quad , \quad Z_{21} = 21 \quad , \quad Z_{22} = 23.4$$

??
 $\therefore Y_{11} = \frac{Z_{22}}{\Delta Z} = \frac{Z_{22}}{Z_{11}Z_{22} - Z_{12}Z_{21}} = \frac{23.4}{25 \times 23.4 - (21)^2}$

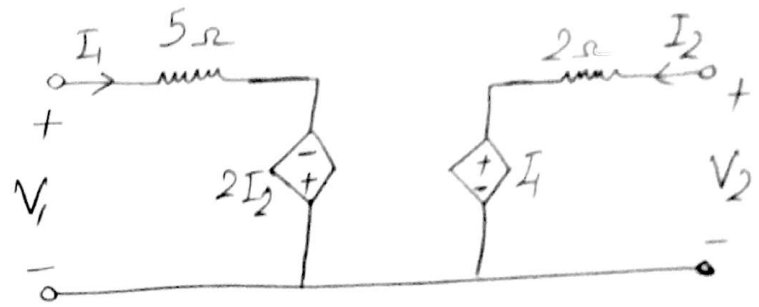
$$Y_{11} = 0.1625$$

$$h_{22} = \frac{1}{Z_{22}} = \frac{1}{23.4} = 0.0427$$

Solved Examples

1

(1) Find Z-Parameters & the corresponding y-Parameters



Solution

⇒ Applying KVL at loop (1):

$$V_1 - 5I_1 + 2I_2 = 0$$

$$\therefore V_1 = 5I_1 - 2I_2$$

⇒ KVL at loop (2):

$$V_2 = 2I_2 + I_1 = I_1 + 2I_2$$

$$Z_{11} = 5$$

$$Z_{12} = -2$$

$$Z_{21} = 1$$

$$Z_{22} = 2$$

⇒ To get y-Param. from Z-Param.:

$$\Delta Z = Z_{11}Z_{22} - Z_{12}Z_{21} = 12$$

$$y_{11} = \frac{Z_{22}}{\Delta Z} = \frac{1}{6}$$

$$y_{12} = \frac{-Z_{12}}{\Delta Z} = \frac{1}{6}$$

$$y_{21} = \frac{-Z_{21}}{\Delta Z} = \frac{-1}{12}$$

$$y_{22} = \frac{Z_{11}}{\Delta Z} = \frac{5}{12}$$

b) To get y-parameters from Z-param. :

3

$$y_{11} = \frac{Z_{22}}{\Delta Z} = \frac{1}{4}$$

$$y_{12} = \frac{-Z_{12}}{\Delta Z} = 0$$

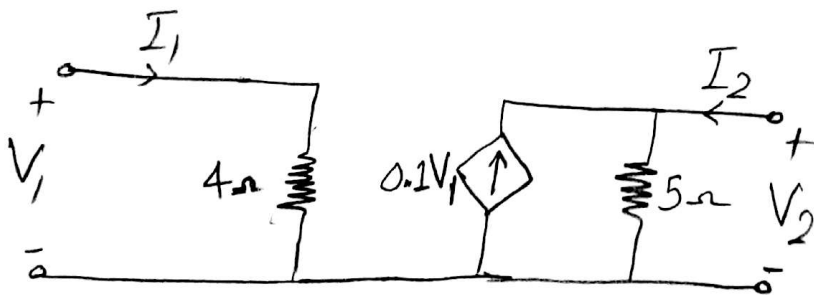
$$y_{21} = \frac{-Z_{21}}{\Delta Z} = \frac{-1}{10}$$

$$y_{22} = \frac{Z_{11}}{\Delta Z} = \frac{1}{5}$$

$$\therefore I_1 = \frac{1}{4} V_1$$

$$I_2 = \frac{-1}{10} V_1 + \frac{1}{5} V_2$$

$\therefore$ The equivalent circuit:

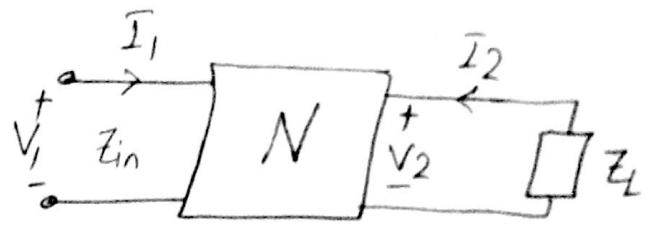


$$(3.) V_1 = \frac{1}{N} V_2$$

$$I_1 = -N I_2$$

a) Find the ABCD-Parameters.

b) Find Z_{in}



Solution

a) For the ABCD-model:

$$V_1 = A V_2 - B I_2$$

$$I_1 = C V_2 - D I_2$$

$$\therefore A = \frac{1}{N}$$

$$, B = 0$$

$$C = 0$$

$$, D = N$$

$$b) Z_{in} = \frac{V_1}{I_1}$$

$$\Rightarrow V_1 = \frac{1}{N} V_2 \rightarrow (1)$$

$$\Rightarrow I_1 = -N I_2 \rightarrow (2)$$

$$\Rightarrow V_2 = -I_2 Z_L \rightarrow (3)$$

$\Rightarrow$ From (3) into (1):

$$\therefore V_1 = -\frac{1}{N} Z_L I_2 \rightarrow (4)$$

$\Rightarrow$ Dividing (4) by (2):

$$\therefore \frac{V_1}{I_1} = \frac{Z_L}{N^2} \Rightarrow Z_{in} = \frac{Z_L}{N^2}$$